Worksheet Answers and Solutions

1. $\int_0^{14} e^{\frac{x}{7}} dx = 7e^{\frac{x}{7}}\Big|_0^{14} = 7\left(e^2 - 1\right).$

2. $\int_0^1 \pi + 3x^2 dx = \left(\pi x + x^3\right)\Big|_0^1 = \pi + 1.$

3. $\int_1^4 \frac{2x^2 + \sqrt{x}}{x} dx = \int_1^4 2x + x^{-\frac{1}{2}} dx = \left(x^2 + 2x^{\frac{1}{2}}\right)\Bigg|_1^4 = 16 + 4 - (1+2) = 17.$

4. Let $u = \frac{3}{t}$; $\frac{du}{dt} = -\frac{3}{t^2}$; $-\frac{1}{3}du = \frac{1}{t^2}dt \Rightarrow$

$$\int_{\frac{\pi}{3}}^{\frac{6}{\pi}} \frac{\sin\left(\frac{3}{t}\right)}{t^2} dt = -\frac{1}{3}\int_{\pi}^{\frac{\pi}{2}} \sin u\, du = \frac{1}{3}\cos u\Big|_{\pi}^{\frac{\pi}{2}} = \frac{1}{3}\left(0 - (-1)\right) = \frac{1}{3}.$$

5. Let $u = \ln x$; $\frac{du}{dx} = \frac{1}{x}$; $du = \frac{1}{x}dx \Rightarrow \int_1^{e^9} \frac{\sqrt{\ln x}}{x} dx = \int_0^9 u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}}\Big|_0^9 = \frac{2}{3}\cdot 27 = 18.$

6. Let $u = \sin x$; $\frac{du}{dx} = \cos x$; $du = \cos x\, dx \Rightarrow \int_0^{\frac{\pi}{4}} \cos x \sin x\, dx = \int_0^{\frac{\sqrt{2}}{2}} u\, du = \frac{u^2}{2}\Big|_0^{\frac{\sqrt{2}}{2}} = \frac{1}{4}.$

(Here you could also choose $u = \cos x$.)

7. $\int_0^{\frac{\pi}{12}} \sec 3x \tan 3x\, dx = \frac{1}{3}\sec 3x\Big|_0^{\frac{\pi}{12}} = \frac{1}{3}\left(\sqrt{2} - 1\right).$

8. Let $u = \sqrt{\frac{w}{4}}$; $\frac{du}{dw} = \frac{1}{8}\left(\frac{w}{4}\right)^{-\frac{1}{2}}$; $8du = \frac{1}{\sqrt{\frac{w}{4}}}dw \Rightarrow$

$$\int_{\pi^2}^{4\pi^2} \frac{\cos\sqrt{\frac{w}{4}}}{\sqrt{\frac{w}{4}}} dw = 8\int_{\frac{\pi}{2}}^{\pi} \cos u\, du = 8\sin u\Big|_{\frac{\pi}{2}}^{\pi} = -8.$$

9. $\int_1^2 \frac{dt}{1+t^2} = \arctan t\Big|_1^2 = \arctan 2 - \frac{\pi}{4}.$

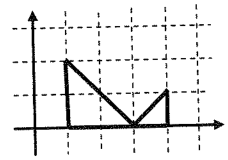
10. Let $u = 4 - t^2$; $\frac{du}{dt} = -2t$; $-\frac{1}{2}du = t\,dt \Rightarrow$

$$\int_{-1}^{0} \frac{t\,dt}{\sqrt{4-t^2}} = -\int_3^4 \frac{du}{2\sqrt{u}} = -\sqrt{u}\Big|_3^4 = -\left(2 - \sqrt{3}\right) = \sqrt{3} - 2.$$

11. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dt}{\sqrt{1-t^2}} = \arcsin t\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$

12. $\int_1^4 |x-3|\,dx = \int_1^3 (3-x)\,dx + \int_3^4 (x-3)\,dx = \left(3x - \frac{x^2}{2}\right)\Big|_1^3 + \left(\frac{x^2}{2} - 3x\right)\Big|_3^4 =$

$\left(9-\frac{9}{2}\right) - \left(3-\frac{1}{2}\right) + (8-12) - \left(\frac{9}{2}-9\right) = \frac{5}{2}$



But a geometric solution is shorter: the integral is equal to the sum of the areas of two right triangles.

13. $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \csc^2(3s)\,ds = -\frac{1}{3}\cot(3s)\Big|_{\frac{\pi}{12}}^{\frac{\pi}{6}} = -\frac{1}{3}\cot\left(\frac{\pi}{2}\right) + \frac{1}{3}\cot\left(\frac{\pi}{4}\right) = \frac{1}{3}$

14. Let $u = 3x - x^2$; $\frac{du}{dx} = 3 - 2x$; $2du = (6-4x)\,dx \Rightarrow$

$\int_{-1}^{2} (3x-x^2)^3 (6-4x)\,dx = 2\int_{-4}^{2} u^3 du = \frac{u^4}{2}\Big|_{-4}^{2} = 8 - 128 = -120$.

15. The key is to recognize that $\frac{1}{\cos^2 p} = \sec^2 p$. Then let

$u = \tan p$; $du = \sec^2 p\ dp \Rightarrow \int_0^{\frac{\pi}{4}} \frac{\tan^4 p}{\cos^2 p}\,dp = \int_0^1 u^4 du = \frac{u^5}{5}\Big|_0^1 = \frac{1}{5}$

16. Let $u = 2 + 4x^2$; $\frac{du}{dx} = 8x$; $\frac{1}{8}du = xdx \Rightarrow$

$\int_{-1}^{0} \frac{x}{(2+4x^2)^2}\,dx = \frac{1}{8}\int_6^2 u^{-2} du = -\frac{1}{8}\cdot\frac{1}{u}\Big|_6^2 = -\frac{1}{8}\left(\frac{1}{2}-\frac{1}{6}\right) = -\frac{1}{24}$

17. Let $u = e^{2x} + 1$; $\frac{du}{dx} = 2e^{2x}$; $\frac{1}{2}du = e^{2x}dx \Rightarrow$

$\int_0^1 \frac{e^{2x}}{e^{2x}+1}\,dx = \frac{1}{2}\int_2^{e^2+1} \frac{du}{u} = \frac{1}{2}\ln u\Big|_2^{e^2+1} = \frac{1}{2}\left(\ln(e^2+1) - \ln 2\right)$

18. $\int_0^{\pi/2} \tan\left(\frac{x}{2}\right)dx = 2\ln\left(\sec\left(\frac{x}{2}\right)\right)\Big|_0^{\pi/2} = 2\left(\ln\sqrt{2}\right) = \ln 2$

19. $\int_0^{e-1} \frac{1}{u+1}\,du = \ln|u+1|\Big|_0^{e-1} = 1$

20. $\int_2^3 e^3\,dt = e^3 t\Big|_2^3 = 3e^3 - 2e^3 = e^3$. Don't be fooled by the format: e^3 is just a constant.

21. With $x = t-1$, $dx = dt$ and $t = x+1$. So,

$\int_1^2 (t-1)^4\,2t\,dt = 2\int_0^1 x^4(x+1)\,dx = 2\int_0^1 x^5 + x^4\,dx$